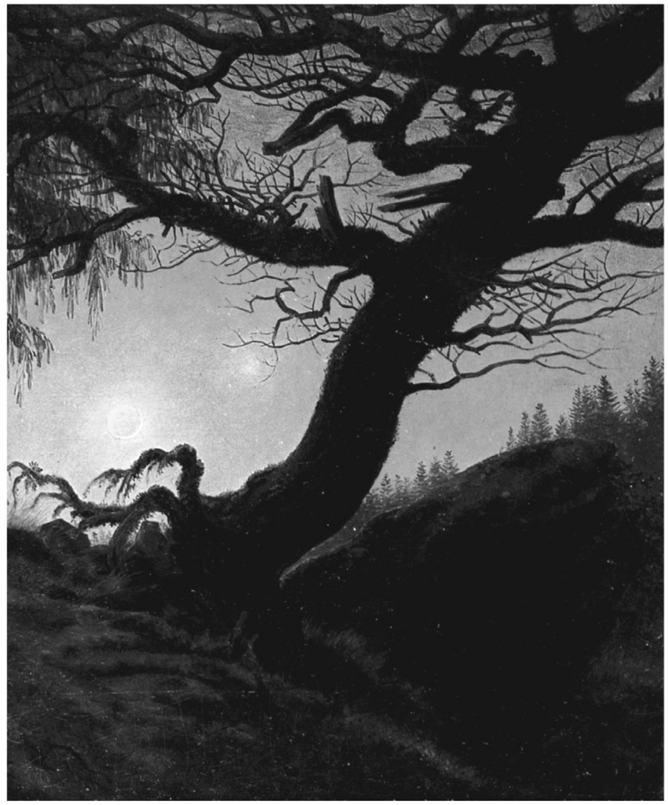
# Combined Stress



from “Man und Frau den Mond betrachtend”
1830-35 by Caspar David Friedrich
Alte Nationalgalerie, Berlin

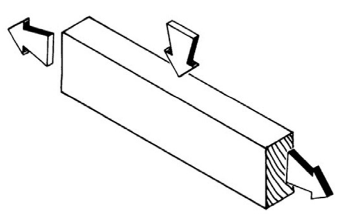
- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas



# NDS - 3.9 Combined Bending and Axial Loading

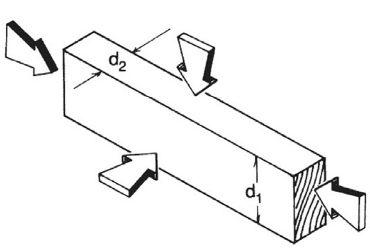
types and applications

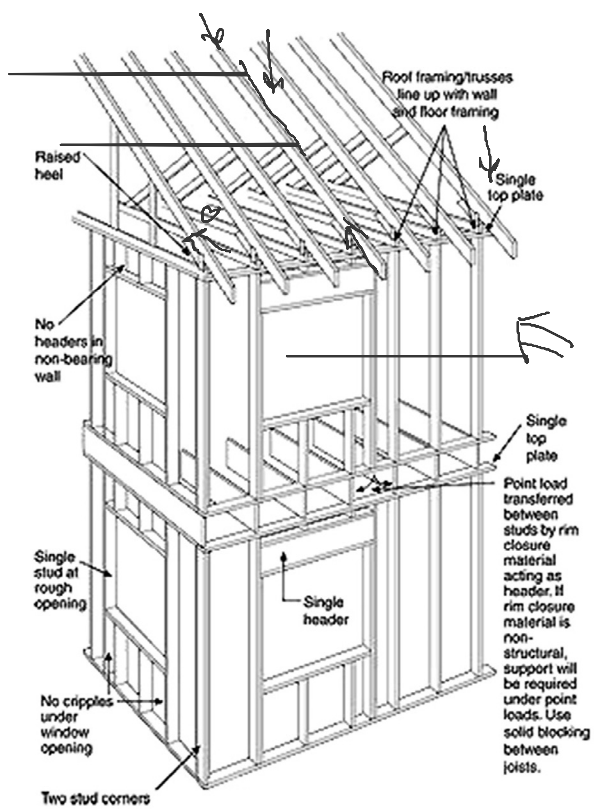
**Figure 3G** **Combined Bending and Axial Tension**



**3.9.2 Bending and Axial Compression**

**Figure 3H** **Combined Bending and Axial Compression**



## NDS 3.9.1 Bending and Axial Tension
two conditions

### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0 \qquad \text{TENSION CRIT.} \qquad (3.9\text{-}1)$$

**and**

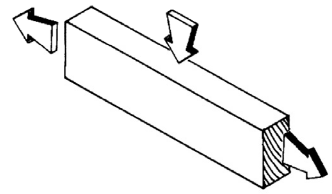
$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0 \qquad \text{FLEXURE CRIT.} \qquad (3.9\text{-}2)$$

**where:**

$F_b^*$ = reference bending design value multiplied by all applicable adjustment factors except $C_L$

$F_b^{**}$ = reference bending design value multiplied by all applicable adjustment factors except $C_V$

**Figure 3G Combined Bending and Axial Tension**





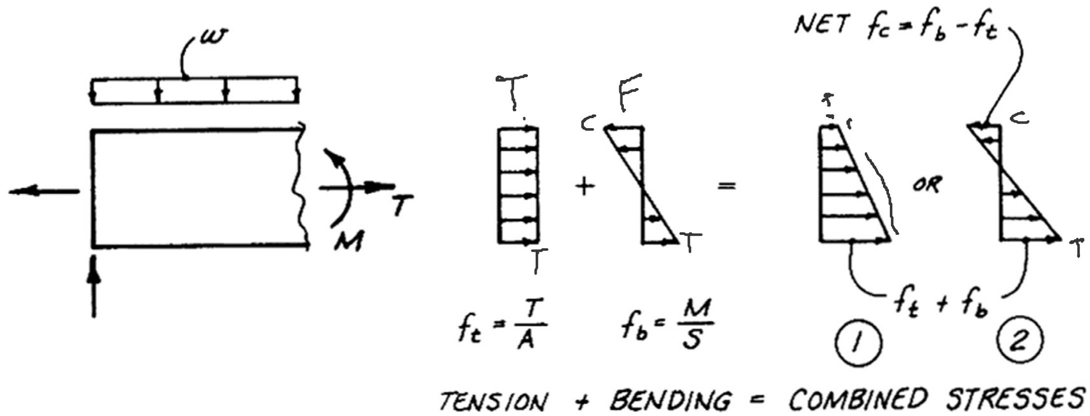
## NDS 3.9.1 Bending and Axial Tension
NDS Equations

CASE 1. Tension is critical. eq. 3.9-1
* no $C_L$

$$\frac{f_t}{F_{t'}} + \frac{f_b}{F_b*} \leq 1.0$$

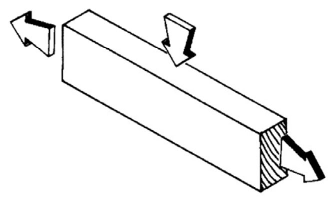
CASE 2. Flexure is critical. eq. 3.9-2
** no $C_V$

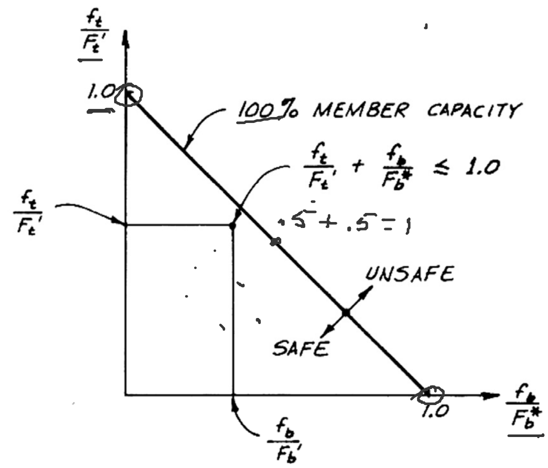
$$\frac{f_b - f_t}{F_b**} \leq 1.0$$



# NDS 3.9.1 Bending and Axial Tension
tension + bending

**Figure 3G Combined Bending and Axial Tension**






BASIC STRAIGHT LINE INTERACTION FORMULA



# NDS 3.9.2 Bending and Axial Compression
two axis bending + compression

**3.9.2 Bending and Axial Compression**

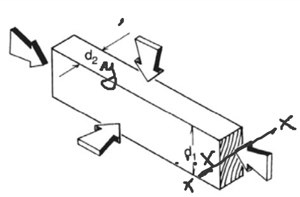
Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3H) shall be so proportioned that:

COMP X-X AMPLIFICATION Y-Y

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1-\left(f_c/F_{cE1}\right)\right]} + \frac{f_{b2}}{F_{b2}'\left[1-\left(f_c/F_{cE2}\right)-\left(f_{b1}/F_{bE}\right)^2\right]} \leq 1.0 \quad (3.9\text{-}3)$$

and **(Flatwise bending + compression)**

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}}\right)^2 < 1.0 \quad (3.9\text{-}4)$$



$$f_c = \frac{P}{A}$$

$$f_b = \frac{M}{S}$$

where:

$f_c < F_{cE1} = \frac{0.822\,E_{min}'}{(\ell_{e1}/d_1)^2}$ for either uniaxial edgewise bending or biaxial bending

and

$f_c < F_{cE2} = \frac{0.822\,E_{min}'}{(\ell_{e2}/d_2)^2}$ for uniaxial flatwise bending or biaxial bending

and

$f_{b1} < F_{bE} = \frac{1.20\,E_{min}'}{(R_B)^2}$ for biaxial bending

$f_{b1}$ = actual edgewise bending stress (bending load applied to narrow face of member), psi

$f_{b2}$ = actual flatwise bending stress (bending load applied to wide face of member), psi

$d_1$ = wide face dimension (see Figure 3H), in.

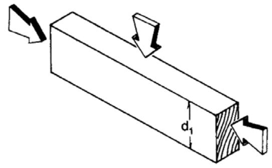
$d_2$ = narrow face dimension (see Figure 3H), in.

# NDS 3.9.2 Bending and Axial Compression

three possible combinations

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1-\left(f_c/F_{cE1}\right)\right]} \le 1.0$$

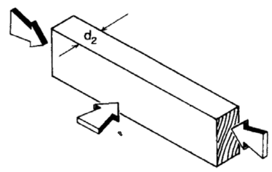
COMP. + FLEXURE X-X




$$f_c < F_{cE1} = \frac{0.822\,E_{min}'}{(\ell_{e1}/d_1)^2}$$

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b2}}{F_{b2}'\left[1-\left(f_c/F_{cE2}\right)-\left(f_{b1}/F_{bE}\right)^2\right]} \le 1.0$$

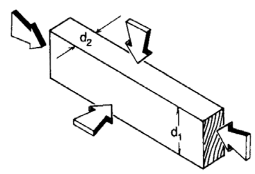
COMP. + FLEXURE Y-Y




$$f_c < F_{cE2} = \frac{0.822\,E_{min}'}{(\ell_{e2}/d_2)^2}$$

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1-\left(f_c/F_{cE1}\right)\right]} + \frac{f_{b2}}{F_{b2}'\left[1-\left(f_c/F_{cE2}\right)-\left(f_{b1}/F_{bE}\right)^2\right]} \le 1.0$$

COMP. + FLEXURE X-X + FLEXURE Y-Y

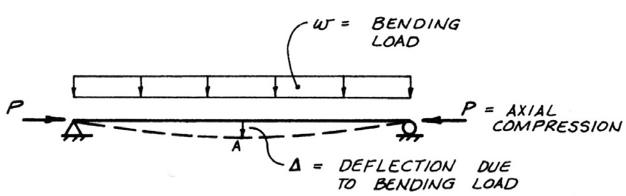


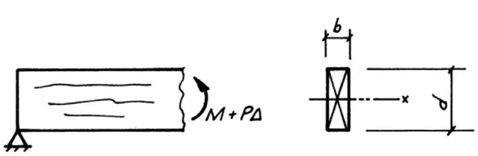

$$f_{b1} < F_{bE} = \frac{1.20\,E_{min}'}{(R_B)^2}$$



# NDS 3.9.2 Bending and Axial Compression

strong axis bending + compression

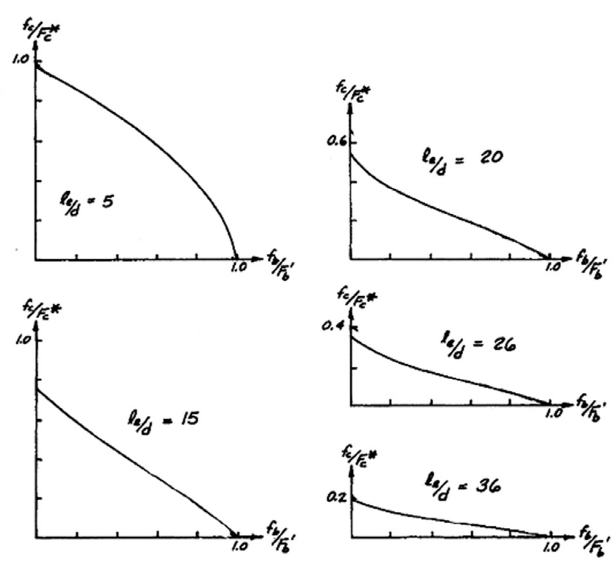



$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'}\ \frac{1}{\left[1-\left(f_c/F_{cE1}\right)\right]} \le 1.0$$

AMPLIFICATION FACTOR

COMP. + FLEXURE X-X



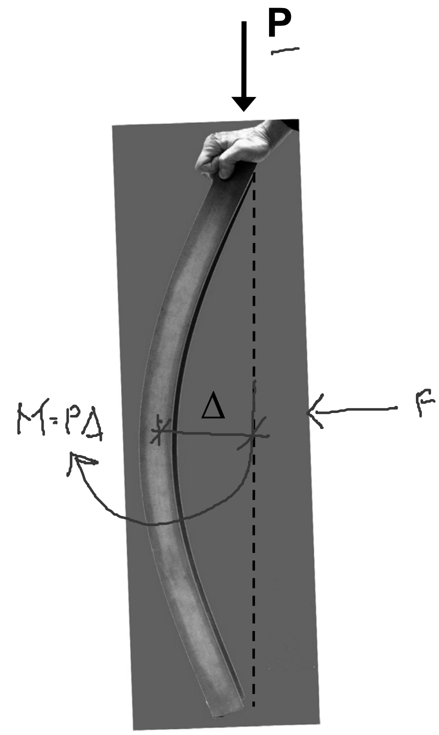

Interaction graphs for different slenderness rations - Breyer

# Second Order Stress
"P Delta Effect"

With larger deflections this can become significant.

1. Eccentric load causes bending moment
2. Bending moment causes deflection, Δ
3. P x Δ causes additional moment

Accounted for by use of an amplification factor



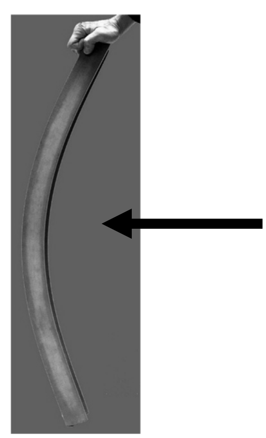

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'}\;\underbrace{\frac{1}{\left[1-\left(f_c/F_{cE1}\right)\right]}}_{\text{AMPLIFICATION FACTOR}} \le 1.0$$

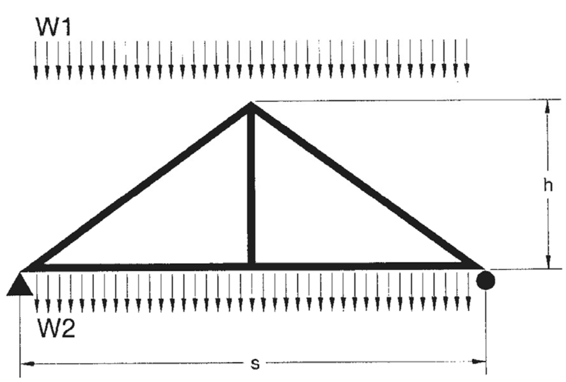
COMP. + FLEXURE X-X



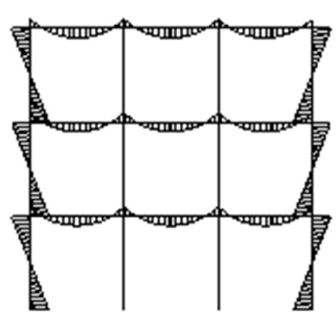
# Examples
combined stress

Columns with side loading





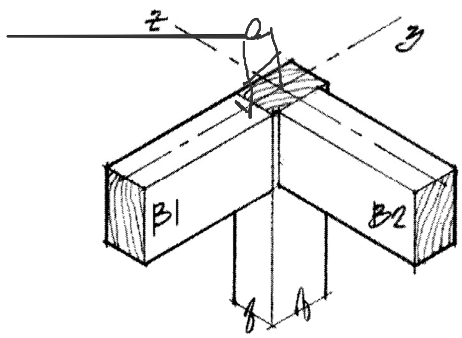

Trusses loaded on members

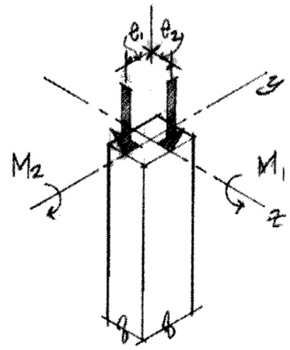


Continuous beams

# Other Examples

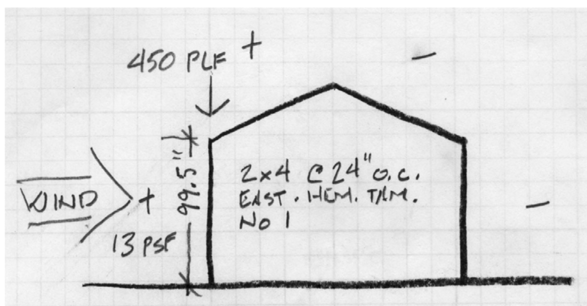
combined stress




$M_1 = P_1 \times e_1$ (ABOUT THE x-AXIS)

$M_2 = P_2 \times e_2$ (ABOUT THE y-AXIS)






# Example Problem

Given: Queen Post truss

Hem-Fir No.1 & Better

$F_b$ = 1100 psi

$F_t$ = 725 psi

$F_c$ = 1350 psi

$E_{min}$ = 550000 psi

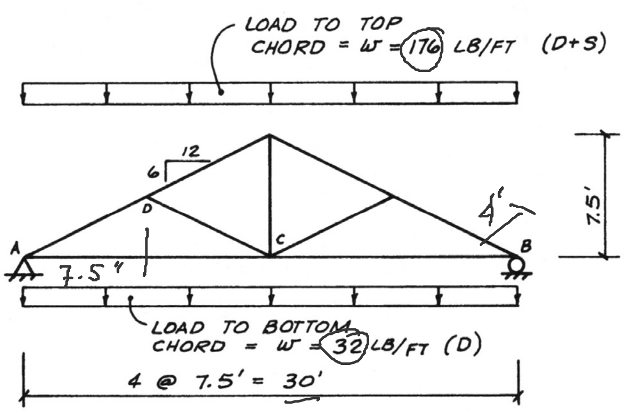
span = 30 ft. spaced 4 ft. o.c.

Projected Roof Load:

D = 14 psf S = 30 psf = 44 psf

Attic + Ceiling:

D = 8 psf

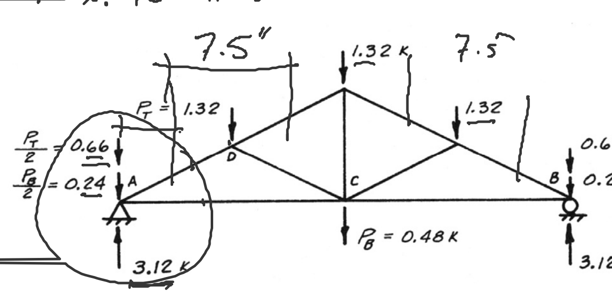



$7.5 \times .176 = 1.32^K$

bottom chord: 2x8 (1.5" x 7.25")

top chord: 2x10 (1.5" x 9.25")

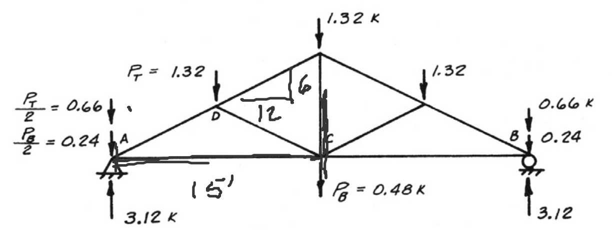
Find: pass/fail



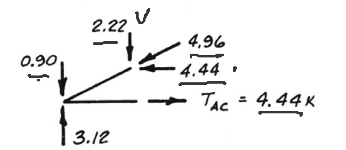

1. Determine truss joint loading

# Example
truss reactions and member forces

2. Determine the external **end reactions** of the whole truss. The geometry and loads are symmetric, so each reaction is ½ of the total load.

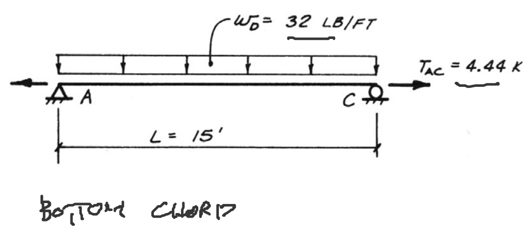



3. Use an FBD of the reaction joint to find the **chord forces**. Sum the forces horizontal and vertical to find the components.




$$\Sigma F_V = -.9 + 3.12 - V = 0$$
$$V = 2.22$$

D+S Load:
Top chord = 4.96 k compression
Bottom chord = 4.44 k tension

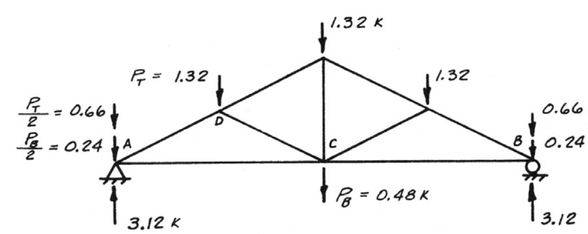




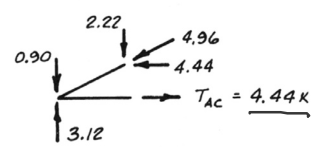

# Example
**bottom chord** 2x8 – bending + tension

D+S

4. Determine controlling load case:

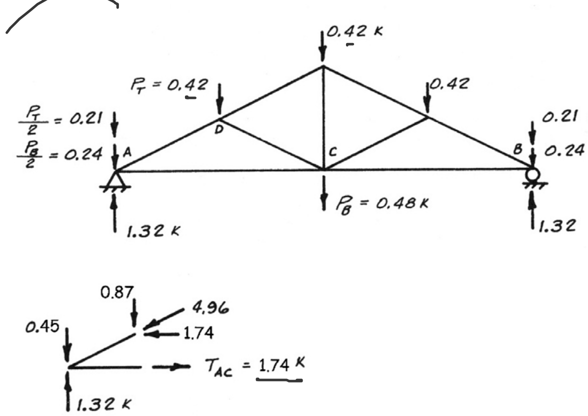



Tension Force: D+S
$P_t / C_D$ = 4.44 lbs / 1.15 = 3.86 (controls)




~~D alone~~

Tension Force: D
$P_t / C_D$ = 1.74 k / 0.9 = 1.93

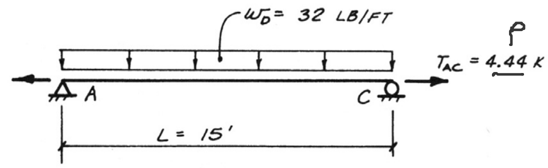


# Example

**bottom chord** 2x8 – bending + tension

$$\frac{f_t}{F_{t'}} + \frac{f_b}{F_b*} \leq 1.0 \qquad \frac{f_b - f_t}{F_b**} \leq 1.0$$

D+S




5. Calculate the **actual** axial and flexural stress.

   $f_t$ = 408.3 psi

   $f_b$ = 821.9 psi

$$f_t = \frac{P}{A} = \frac{4440 \text{ lbs}}{10.875 \text{ in}^2} = 408.3 \text{ psi}$$

2x8

$$f_b = \frac{M}{S_x} = \frac{900(12)}{13.14} = 821.9 \text{ psi}$$

$$M = \frac{w l^2}{8} = \frac{32 (15)^2}{8} = 900 \text{ '-\#}$$

PLF

$$S_x = 13.14 \text{ in}^3$$

2x8



# Example

**bottom chord** 2x8 – bending + tension

Hem-Fir No.1 & Better
$F_b$ = 1100 psi
$F_t$ = 725 psi

$C_L$ IS 1.0 BY 4.4.1
d/b = 4, ENDS ARE HELD

6. Determine **allowable** stresses using applicable factors:

   Tension: D+S
   $F_t' = F_t (C_D\ C_F)$
   $F_t'$ = 725 (1.15 1.2) = 1000 psi

   $F_t'$ = 1000 > 408.3 = $f_t$ **ok**

   Flexure: D+S
   $F_b' = F_b (C_D\ C_L\ C_F)$
   $F_b'$ = 1100 (1.15 1.0 1.2) = 1518 psi

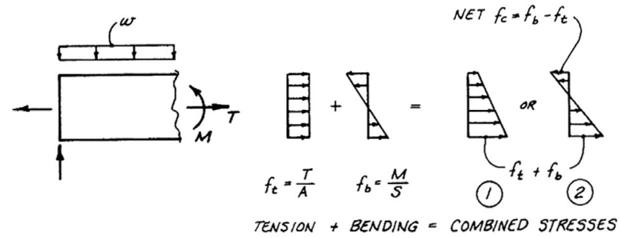
   $F_b'$ = 1518 > 821.9 = $f_b$ **ok**

**Size Factors, $C_F$**

| Grades | Width (depth) | $F_b$ Thickness (breadth) 2" & 3" | $F_b$ Thickness (breadth) 4" | $F_t$ | $F_c$ |
|---|---|---|---|---|---|
| Select Structural, No.1 & Btr, No.1, No.2, No.3 | 2", 3", & 4" | 1.5 | 1.5 | 1.5 | 1.15 |
| | 5" | 1.4 | 1.4 | 1.4 | 1.1 |
| | 6" | 1.3 | 1.3 | 1.3 | 1.1 |
| | 8" | $\boxed{1.2}$ | 1.3 | $\boxed{1.2}$ | 1.05 |
| | 10" | 1.1 | 1.2 | 1.1 | 1.0 |
| | 12" | 1.0 | 1.1 | 1.0 | 1.0 |
| | 14" & wider | 0.9 | 1.0 | 0.9 | 0.9 |
| Stud | 2", 3", & 4" | 1.1 | 1.1 | 1.1 | 1.05 |
| | 5" & 6" | 1.0 | 1.0 | 1.0 | 1.0 |
| | 8" & wider | Use No.3 | | | |
| Construction Standard | 2", 3", & 4" | 1.0 | 1.0 | 1.0 | 1.0 |
| Utility | 4" | 1.0 | 1.0 | 1.0 | 1.0 |
| | 2" & 3" | 0.4 | — | 0.4 | 0.6 |

# Example

**bottom chord** 2x8 – combined stress




### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \le 1.0 \qquad \text{TENSION CRIT.} \qquad (3.9\text{-}1)$$

and

$$\frac{f_b - f_t}{F_b^{**}} \le 1.0 \qquad \text{FLEXURE CRIT.} \qquad (3.9\text{-}2)$$

**where:**

$F_b^*$ = reference bending design value multiplied by all applicable adjustment factors except $C_L$

$F_b^{**}$ = reference bending design value multiplied by all applicable adjustment factors except $C_V$ .

(3.9-1)

$$\frac{f_t}{F_t}\;\frac{408.3}{1000} + \frac{821.9}{1518}$$

$0.4083 + 0.5414 = 0.95$

$0.95 < 1.0$ ✓PASS

(3.9-2)

$$\frac{821.9 - 408.3}{1518} = 0.2724$$

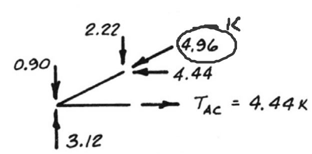
$0.27 < 1.0$ ✓PASS

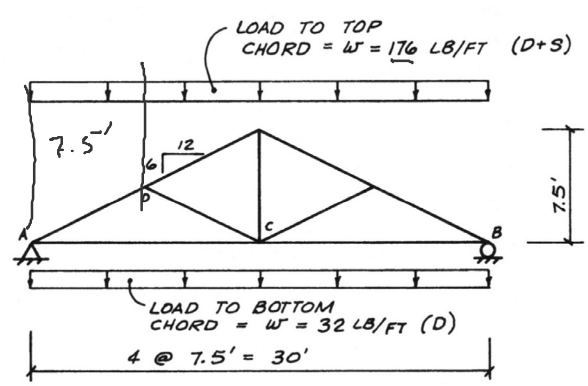


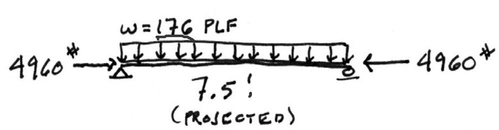
# Example

**top chord** 2x10 – bending + compression

D+S




$$M = \frac{w\ell^2}{8} = \frac{176\text{ PLF}(7.5')^2}{8} = 1237.5'\text{-}\#$$

$S_x = 21.39\text{ in}^3$ 2x10

4. Calculate the **actual** axial and flexural stress.

$f_c$ = 357.5 psi

$$f_c = \frac{P}{A} = \frac{4960\#}{1.5 \times 9.25} = 357.5\text{ psi}$$

$f_b$ = 694.2 psi

$$f_b = \frac{M}{S_x} = \frac{1237.5\,(12)}{21.39} = 694.2\text{ psi}$$

# Example

**top chord** 2x10 – bending + compression

Hem-Fir No.1 & Better
$F_b$ = 1100 psi
$F_c$ = 1350 psi
$E_{min}$ = 550000 psi

5. Determine **allowable** stresses using applicable factors:

(**compression**: D+S)
$F_c' = F_c\ (C_D\ C_F\ C_P)$
$F_c'$ = 1350 (1.15 1.0 0.897 ) = 1392.7 psi > 357.5

(**flexure**: D+S)
$F_b' = F_b\ (C_D\ C_L\ C_F)$
$F_b'$ = 1100 (1.15 1.0 1.1) = 1392 psi > 694.2
S

strong axis buckling

$C_P$
$\ell_e = 8.385'$ $d = 9.25''$
$\ell_e/d = \frac{8.385(12)}{9.25} = 10.88$
$F_{cE} = \frac{0.822\ E_{min}'}{(\ell_e/d)^2} = \frac{0.822\ (550000)}{10.88^2} = 3820$ psi
$F_c^* = 1350\ (1.15\ 1.0) = 1552.5$ psi
$F_{cE}/F_c^* = \frac{3820}{1552} = 2.46$ $c = 0.8$
$C_P = 0.897$

Size Factors, $C_F$

| Grades | Width (depth) | $F_b$ Thickness (breadth) 2" & 3" | $F_b$ Thickness (breadth) 4" | $F_t$ | $F_c$ |
|---|---|---|---|---|---|
| Select Structural, No.1 & Btr, No.1, No.2, No.3 | 2", 3", & 4" | 1.5 | 1.5 | 1.5 | 1.15 |
| | 5" | 1.4 | 1.4 | 1.4 | 1.1 |
| | 6" | 1.3 | 1.3 | 1.3 | 1.1 |
| | 8" | 1.2 | 1.3 | 1.2 | 1.05 |
| | 10" | $\boxed{1.1}$ | 1.2 | 1.1 | $\boxed{1.0}$ |
| | 12" | 1.0 | 1.1 | 1.0 | 1.0 |

$C_L$
The top chord is braced by the plywood sheathing so $C_L$ = 1.0



# Example

top chord 2x10 Eq. 3.9-3

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1-(f_c/F_{cE1})\right]} \le 1.0$$

COMP. + FLEXURE X-X

$$F_{cE1} = \frac{0.822(550000 \text{ psi})}{(100.6''/9.25'')^2} = 3820.7 \text{ psi}$$

**where:**

$$f_c < F_{cE1} = \frac{0.822\,E_{min}'}{(\ell_{e1}/d_1)^2}$$

EULER 1
for either uniaxial edge-wise bending or biaxial bending

**and**

$$f_c < F_{cE2} = \frac{0.822\,E_{min}'}{(\ell_{e2}/d_2)^2}$$

EULER 2
for uniaxial flatwise bending or biaxial bending

**and**

$$f_{b1} < F_{bE} = \frac{1.20\,E_{min}'}{(R_B)^2}$$

LTB
for biaxial bending

$f_{b1}$ = actual edgewise bending stress (bending load applied to narrow face of member)
$f_{b2}$ = actual flatwise bending stress (bending load applied to wide face of member)
$d_1$ = wide face dimension (see Figure 3H)
$d_2$ = narrow face dimension (see Figure 3H)

COMPRESSION:

$$\left[\frac{f_c}{F_c'}\right]^2 = \left[\frac{357.5}{1392.7}\right]^2 = 0.0659$$

FLEXURE:

$$\frac{f_{b1}}{F_{b1}'} = \frac{694.2}{1392} = 0.4987$$

AMPLIFICATION FACTOR:

$$\frac{1}{1-(357.5/3820)} = \frac{1}{0.906}$$

$F_c'$

0.4987 (1.103) = 0.550

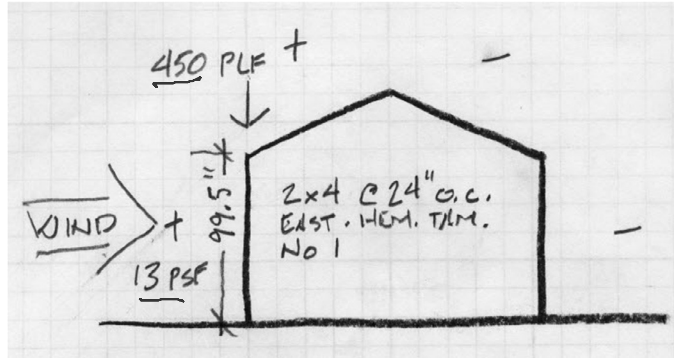
COMBINATION:

0.0659 + 0.550 = 0.616
0.616 < 1.0 ✓ PASS

# Combined Stress in NDS
Stud wall example

Exterior stud wall under bending + axial compression

1. Determine load per stud
2. Use axial load and moment to find actual stresses fc and fb
3. Determine load factors
4. Calculate factored stresses
5. Check NDS equation 3.9-3

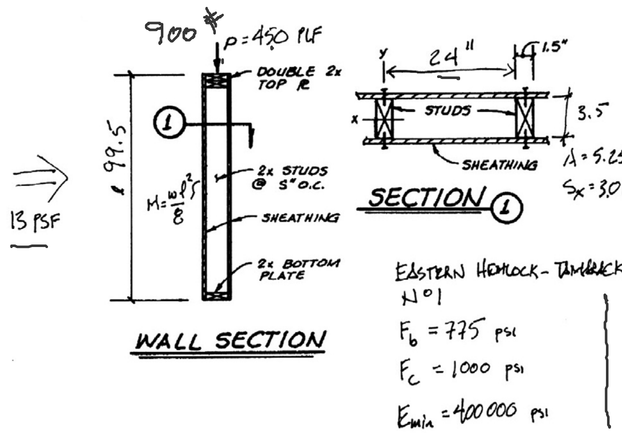



$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'\left[1-\left(f_c/F_{cE1}\right)\right]} \leq 1.0 \quad (3.9\text{-}3)$$

and

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}}\right)^2 < 1.0 \quad (3.9\text{-}4)$$

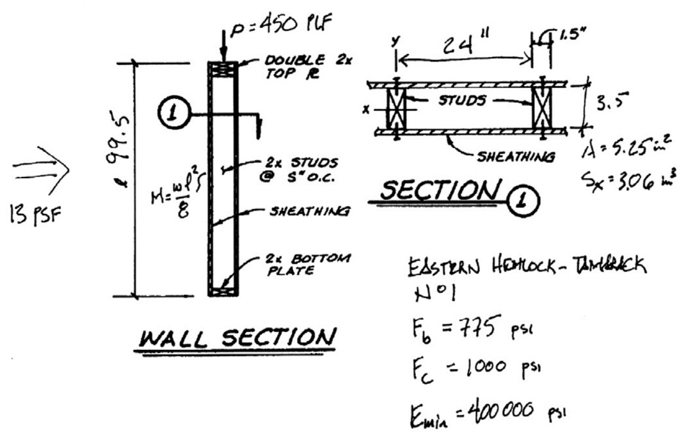
**flatwise bending**






# Combined Stress in NDS
stud wall

Exterior stud wall under bending + axial compression




1. Determine load per stud

$P$ = LOAD/STUD

$$P = 450 \text{ PLF } \frac{OC}{12} = 450\,\frac{24}{12} = \boxed{900 \text{ LBS}}$$

$$w = 13\text{PSF } \frac{OC}{12} = 13\,\frac{24}{12} = 26 \text{ PLF/STUD}$$

2. Use axial load and moment to find actual stresses $f_c$ and $f_b$

$$M_x = \frac{w\ell^2}{8} = \frac{26\,(99.5/12)^2}{8} = \boxed{223.4 \text{ '-\#}}$$

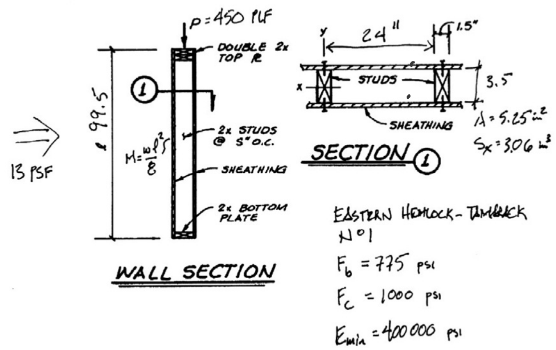
$$f_c = \frac{P}{A} = \frac{900}{5.25} = \boxed{171.43 \text{ PSI}}$$

2x4

$$f_b = M/S_x = \frac{223.4\,(12)}{3.06 \text{ in}^3} = \boxed{875.5 \text{ PSI}}$$

# Combined Stress in NDS
stud wall - bending

Exterior stud wall under
bending + axial compression




**Size Factors, $C_F$**

| Grades | Width (depth) | $F_b$ Thickness (breadth) 2" & 3" | $F_b$ Thickness (breadth) 4" | $F_t$ | $F_c$ |
|---|---|---|---|---|---|
| Select Structural, No.1 & Btr, No.1, No.2, No.3 | 2", 3", & 4" | $\boxed{1.5}$ | 1.5 | 1.5 | $\boxed{1.15}$ |
| | 5" | 1.4 | 1.4 | 1.4 | 1.1 |
| | 6" | 1.3 | 1.3 | 1.3 | 1.1 |
| | 8" | 1.2 | 1.3 | 1.2 | 1.05 |
| | 10" | 1.1 | 1.2 | 1.1 | 1.0 |
| | 12" | 1.0 | 1.1 | 1.0 | 1.0 |

3. Determine load factors (bending)

TABULATED STRESS:

$F_b = 775$ psi $\quad F_c = 1000$ psi $\quad E_{min} = 400\,000$ psi

FACTORS:

$C_D = \boxed{1.6}$ (WIND)

$C_F = \boxed{1.5}$ (FOR $F_b$) $\quad \boxed{1.15}$ (FOR $F_c$)

$C_L = \boxed{1.0}$ (BRACED BY SHEATHING)

$C_r = \boxed{1.15}$ ($\leq 24''$ o.c.)



# Combined Stress in NDS
stud wall - bending

Exterior stud wall under
bending + axial compression

$F_b = 775$ psi

4. Calculate factored stresses

$C_D = 1.6 \qquad C_F = 1.5$

$C_M = 1.0 \qquad C_{fu} = 1.0$

$C_t = 1.0 \qquad C_i = 1.0$

$C_L = 1.0 \qquad C_r = 1.15$

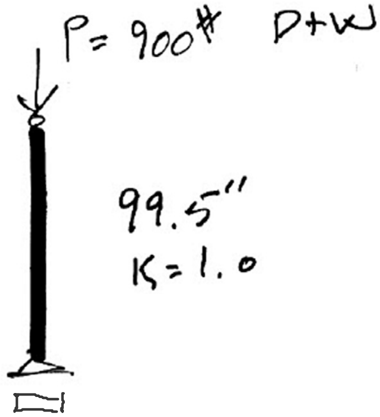
Bending Stress

$$F'_b = 775\underset{C_D}{(1.6)}\underset{C_F}{(1.5)}\underset{C_r}{(1.15)} = 2139 \text{ psi}$$

# Combined Stress in NDS
stud wall - compression

Exterior stud wall under
bending + axial compression

$$C_P = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$




$C_P$ (strong axis)

$F^* = 1000(1.6)(1.15) = 1840$

$F_{cE} = \frac{0.822(400000)}{(99.5/3.5)^2} = 406.8$

$c = 0.8$

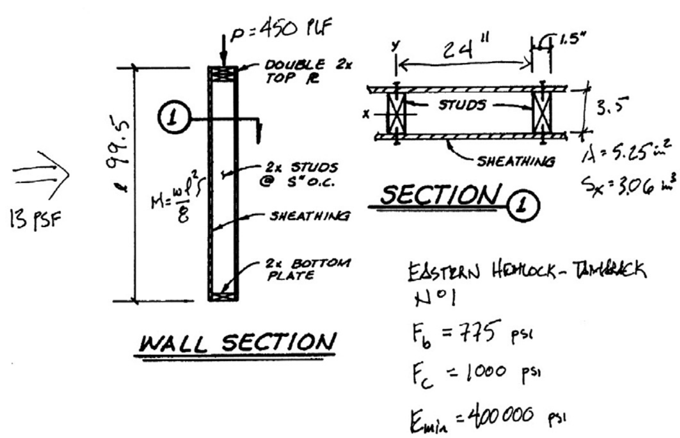
$C_P = 0.21$

3. Determine load factors
(compression - braced in weak axis)



# Combined Stress in NDS
stud wall - compression

Exterior stud wall under
bending + axial compression




4. Calculate factored stresses
$F_c' = F_c\,(C_D\, C_F\, C_P)$

$C_D = 1.6$
$C_F = 1.15$
$C_P = 0.21$

Compression Stress

check fc < F'c

(this should actually be checked for both W+D and D alone)

**Actual Stress**

$f_c = \frac{P}{A} = \frac{900^{\#}}{5.25} = 171.4$ psi

**Factored Allowable Stress**

$F_c' = 1000\underset{C_D}{(1.6)}\underset{C_F}{(1.15)}\underset{C_P}{(0.21)} = 386.4$ psi

# Combined Stress in NDS
stud wall – combined stress

Exterior stud wall under
bending + axial compression

COMP + FLEX (AMP. FAC.)

Combined Stress Calculation

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'} \cdot \frac{1}{1-\left(f_c/F_{cE1}\right)} \lesssim 1.0$$

$$\left[\frac{171.4}{386.4}\right]^2 + \frac{876}{2139} \cdot \frac{1}{1-\left(171.4/406.8\right)}$$

$$0.1967 + (0.4095)(1.728) =$$

$$0.1967 + 0.7077 = 0.9045 \lesssim 1.0 \checkmark \text{ok}$$



# Rafters
Flexure y-y + Axial Compression

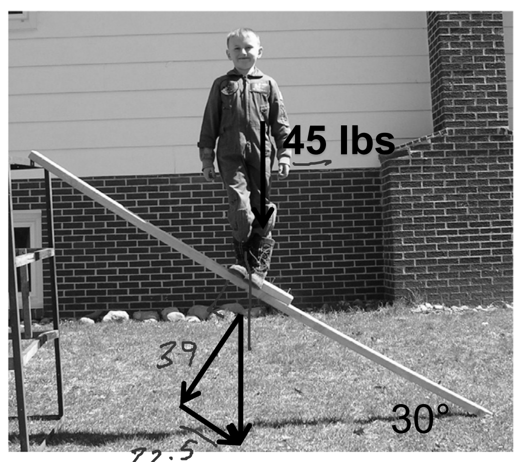
Given:

S-P-F No.2 2x4 x 96”
$F_b$ = 875 psi
$F_c$ = 1150 psi $E_{min}$ = 510000 psi
P = 45 lbs.

Find: pass/fail

Find normal and axial components of the load.

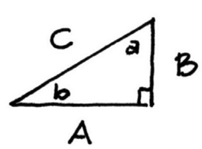
Francesco on 2x4




$L_v$ = 48”

$L_h$ = 83.14”

Axial = 22.5 lbs
Normal = 39 lbs

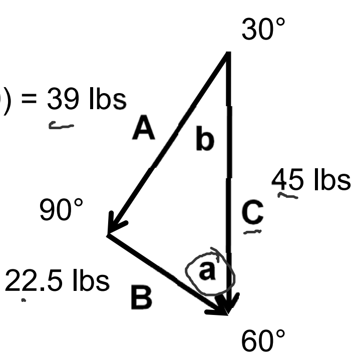



$C = \sqrt{A^2 + B^2}$

$A = C \cos b$
$A = C \sin a$
$B = C \sin b$
$B = C \cos a$

45 (sin60) = 39 lbs

45 (cos60) = 22.5 lbs



# Rafters
Flexure y-y + Axial Compression

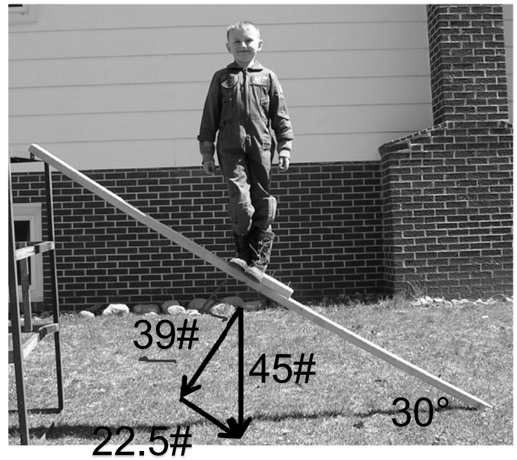
Given:

- S-P-F No.2 2x4 x 96”
- $F_b$ = 875 psi
- $F_c$ = 1150 psi $E_{min}$ = 510000 psi
- P = 45 lbs. (as roof Lr)

Find: pass/fail

Axial = 22.5 lbs
Normal = 39 lbs

Francesco on 2x4




Actual stress:

COMPRESSION:

$$f_c = P/A = \frac{22.5}{5.25} = 4.285 \text{ psi}$$

2x4

$A = 5.25 \text{ in}^2$

$P = 22.5$

FLEXURE Y-Y

$$f_{b2} = \frac{M}{S_y} = \frac{936}{1.313} = 712.8 \text{ psi}$$

$$M = \frac{PL}{4} = \frac{39(96")}{4} = 936 \text{ "-\#}$$

$S_y = 1.313 \text{ in}^3$



# Rafters
Flexure y-y + Axial Compression

S-P-F No.2 2x4 x 96” $F_b$ = 875 psi
$F_c$ = 1150 psi $E_{min}$ = 510000 psi

Determine factored allowable stresses:

**Flat Use Factor, $C_{fu}$**

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, $F_b$, shall also be permitted to be multiplied by the following flat use factors:

**Flat Use Factors, $C_{fu}$**

| Width (depth) | Thickness (breadth) 2" & 3" | 4" |
|---|---|---|
| 2" & 3" | 1.0 | — |
| 4" | 1.1 | 1.0 |
| 5" | 1.1 | 1.05 |
| 6" | 1.15 | 1.05 |
| 8" | 1.15 | 1.05 |
| 10" & wider | 1.2 | 1.1 |

Compression

$F_c' = F_c(C_D C_F C_P)$

$F_c = 1150$ psi

$C_D(L_r) = 1.25$

$C_F = 1.15$

$$F_{cE} = \frac{0.822 E_{min}'}{(l_e/d)^2} = \frac{0.822(510000)}{(96/1.5)^2} = 102.3 \text{ psi}$$

$F_c^* = 1150(1.25\ 1.15) = 1653$ psi

$$F_{cE}/F_c^* = \frac{102.3}{1653} = 0.06191$$

$C_P = 0.0611$

$F_c' = 1150(1.25\ 1.15\ 0.0611) = 101.0$ psi

Flexure y-y

$F_{b2}' = F_b(C_D C_L C_F C_{fu})$

$F_b = 875$ psi

$C_D(L_r) = 1.25$

$C_F = 1.5$

$C_{fu} = 1.1$

$C_L = 1.0$

$F_{b2}' = 875(1.25\ 1.0\ 1.5\ 1.1) = 1804.6$ psi

# Rafters
Flexure y-y + Axial Compression

Eq. 3.9-3

Check combination stresses:

$f_c$ = 4.258 psi
$F_c'$ = 101.0 psi
$f_{b2}$ = 712.8 psi
$F_{b2}'$ = 1804.6 psi
$F_{cE2}$ = 102.3 psi

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b2}}{F_{b2}'\left[1-(f_c/F_{cE2})-(f_{b1}/F_{bE})^2\right]} \leq 1.0$$

COMP. + FLEXURE Y-Y

Compression:

COMPRESSION:

$$\left(\frac{f_c}{F_c'}\right)^2 = \left(\frac{4.285}{101.0}\right)^2 = \underline{0.00180}$$



# Rafters
Flexure y-y + Axial Compression

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b2}}{F_{b2}'\left[1-(f_c/F_{cE2})-(f_{b1}/F_{bE})^2\right]} \leq 1.0$$

COMP. + FLEXURE Y-Y

Check combination stresses:

Flexure y-y

$f_c = 4.285$ psi

$F_{cE2} = 102.3$ psi

$$\frac{f_c}{F_{cE2}} = \frac{4.285}{102.3} = 0.04188$$

$$f_{b1} = \frac{\cancel{M}}{S_x} = \frac{0}{3.06} = 0$$

(in this example there is no strong axis bending, so the term is zero)

FLEXURE Y-Y

$$\frac{f_{b2}}{F_{b2}'} = \frac{712.8}{1804.6} = \underline{0.395}$$

AMPLIFICATION FACTOR:

$$\frac{1}{1-(f_c/F_{cE2})-\cancel{(f_{b1}/F_{bE})^2}}$$

$$\frac{1}{1-\left(\frac{4.285}{102.3}\right)-\cancel{\left(\frac{0}{F_{bE}}\right)^2}} = \frac{1}{1-0.0418-0}$$

$$\frac{1}{0.9582} = \underline{1.043}$$

$$0.395(1.043) = \underline{0.412}$$

# Rafters

## Flexure y-y + Axial Compression

Check combination stresses:

Eq. 3.9-3

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b2}}{F_{b2}'\left[1-(f_c/F_{cE2})-(f_{b1}/F_{bE})^2\right]} \le 1.0$$

COMP. + FLEXURE Y-Y

COMPRESSION + FLEXURE Y-Y

$0.0018 + \underline{0.412} = 0.414$

$0.414 < 1.0$ ✓OK



# Rafters

## Flexure y-y + Axial Compression

Check combination stresses:

Eq. 3.9-4

EQ 3.9-4 FLATWISE BENDING

$$\frac{f_c}{F_{cE2}} + \left(\frac{\overset{0}{f_{b1}}}{F_{bE}}\right)^2 < 1.0$$

$$\frac{4.285}{102.3} + \cancel{\left(\frac{0}{F_{bE}}\right)^2}$$

$0.0419 < 1.0$ PASS ✓