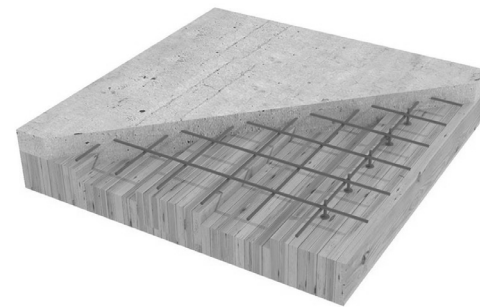Architecture 544
Wood Structures
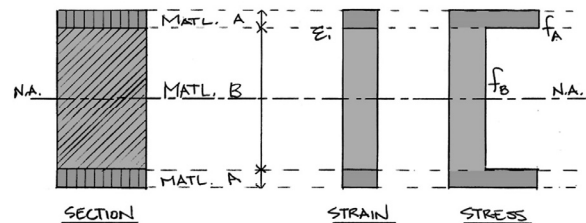
Wood Composite Systems

- Strain Compatibility

- Transformed Sections

- Flitched Beams

- Steel Sandwiched Beams

- Wood – Concrete Composites

Strain Compatibility

With two materials bonded together, both will act as one, and the deformation in each is the same.
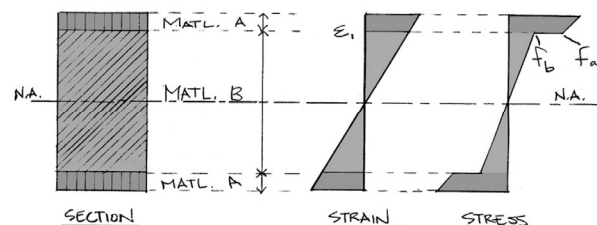
Therefore, the strains will be the same in each material under **axial load**.

Axial

In **flexure** the strains are the same as in a homogeneous section, i.e. linear.

In flexure, if the two materials are at the same distance from the N.A., they will have the same strain at that point because both materials share the same strain diagram. We say the strains are "compatible".
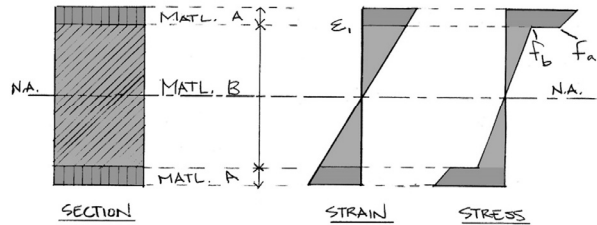
Stress = E x Strain
So stress will be higher if E is higher.
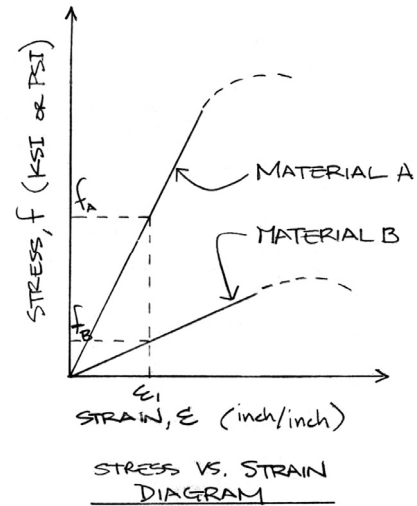

Flexure

Strain Compatibility (cont.)

The stress in each material is determined by using Young's Modulus

SECTION   STRAIN   STRESS

flexure

$$\sigma = \mathrm{E}\varepsilon$$

Care must be taken that the elastic limit of each material is not exceeded. The elastic limit can be expressed in either stress or strain.


STRESS VS. STRAIN DIAGRAM

Axial Compression
Stress Analysis Procedure

Determine the safety (pass or fail) of the composite short pier under an axial load.

By Considère's Law:

1. P = f A = fs As + fw Aw
   (the actual stresses are unknown)

2. ∈pier = ∈s = ∈w (equal strains)
   ∈ = f / E
   fs / Es = fw / Ew
   fs = (Es / Ew) fw

3. Substitute [(Es / Ew) fw] for fs into the original equation and solve for fw

4. Use second equation to solve fs

SECTION   STRAIN   STRESS
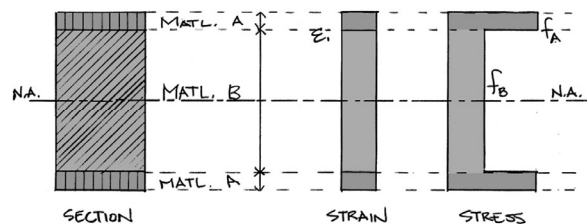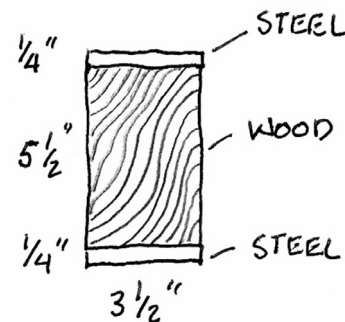
Example - Axial Compression

Pass/Fail Analysis:

Given: Load = 50 kips
  Section: Aw = 19.25 in^2
        Ew = 700 ksi
        As = 2x 0.875 = 1.75 in^2
        Es = 29000 ksi
  Braced against buckling.
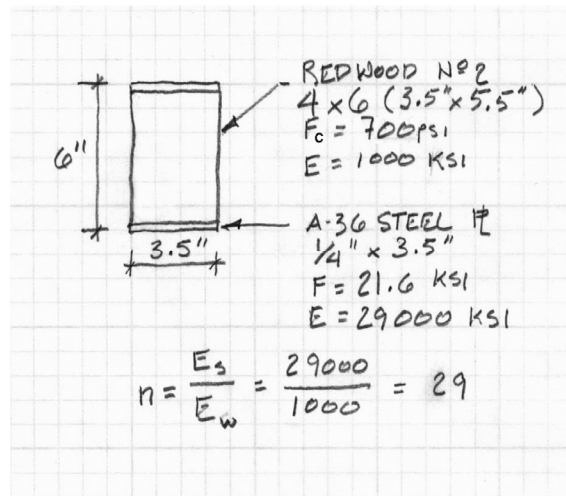
Req'rd: actual stress in the material

1. P = f A = fs As + fw Aw
    (the actual stresses are unknown)

2. fs = (Es / Ew) fw

3. P = (Es / Ew) fw As + fw Aw

REDWOOD N°2
4 x 6 (3.5" x 5.5")
F_c = 700 psi
E = 1000 KSI

A-36 STEEL ⌐
¼" x 3.5"
F = 21.6 KSI
E = 29000 KSI

$$n = \frac{E_s}{E_w} = \frac{29000}{1000} = 29$$

$$P = fA$$
$$50^k = f_s (1.75 in^2) + f_w (19.25)$$
$$f_s = 29 (f_w)$$
$$50 = 29 f_w (1.75) + f_w 19.25$$
$$50 = 70 f_w$$
$$f_w = 0.714 \, K/in^2 = 714 \, PSI$$
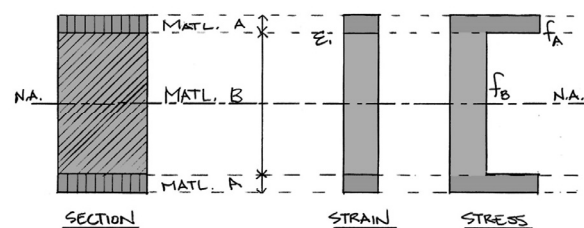

Example - Axial Compression
Pass/Fail Analysis:

4. Use second equation to solve for fs

$$f_s = 29 (f_w)$$
$$f_s = 29 (.714) = 20.7 \, KSI$$

STEEL
 $F_c \geqslant f_c$
  21.6 > 20.7 KSI OK

WOOD
  $F_c \geqslant f_c$
  700 < 714 PSI Fails!

Flitched Beams & Scab Plates
**Advantages**

- Compatible with the wood structure,
    i.e. can be nailed
- Easy to retrofit to existing structure
- Lighter weight than a steel section
- Stronger than wood alone
    – Less deep than wood alone
    – Allow longer spans
- The section can vary over the length of the span to optimize the member (e.g. scab plates)
- The wood stabilizes the thin steel plate
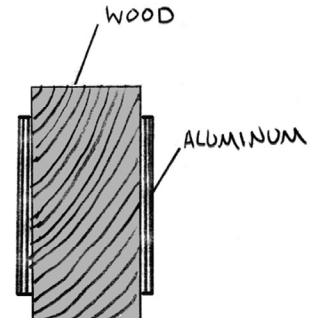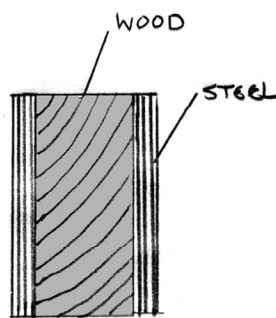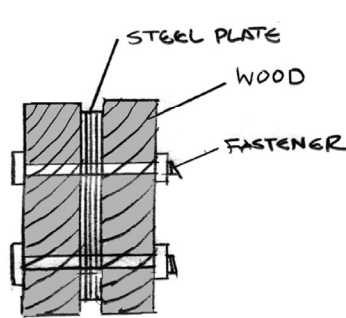
Flitched Beams & Scab Plates
Disadvantages

- More labor to make – expense. Flitched beams requires shop fabrication or field bolting.
- Often replaced by Composite Lumber which is simply cut to length – less labor
    – Glulam
    – LVL
    – PSL
- Flitched Beams are generally heavier than Composite Lumber
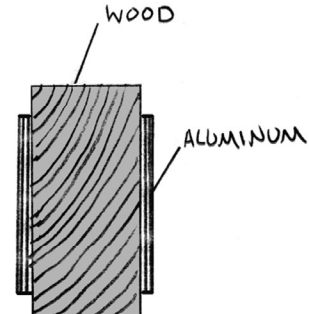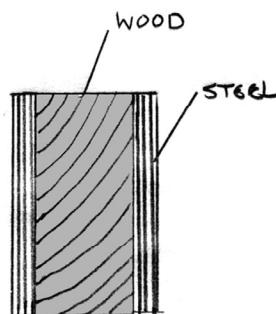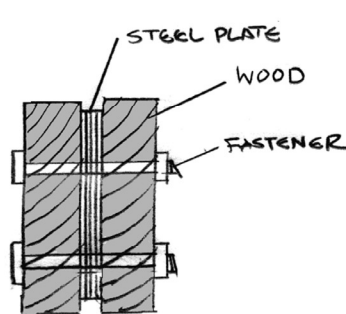
Flexure Stress using Transformed Sections

In the basic flexural stress equation, I is derived based on a homogeneous section. Therefore, to use the stress equation one needs to "transform" the composite section into a homogeneous section.
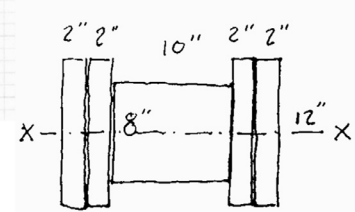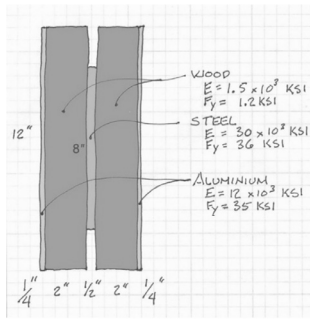
$$f_b = \frac{Mc}{I}$$

Homogeneous Section

$$f_b = \frac{Mc\ n}{I_{tr}}$$

Transformed Section

For the new "transformed section" to behave like the actual section, the stiffness of both would need to be the same.

Since Young's Modulus, E, represents the material stiffness, when transforming one material into another, the area of the transformed material must be scaled by the ratio of one E to the other.

The scale factor is called the modular ratio, n.

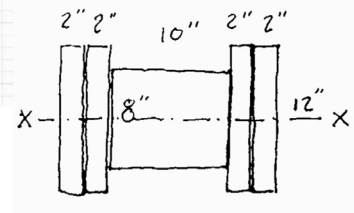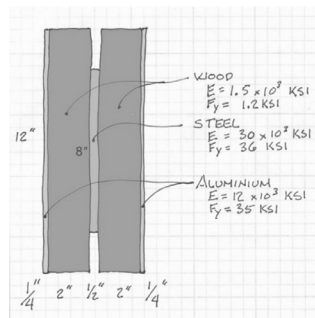$$n = \frac{E_A}{E_B}$$

Transformed material

Base material

In order to also get the correct stiffness for the moment of Inertia, I, only the width of the geometry is scaled. Using I from the transformed section (I_{TR}) will then give the same flexural stiffness as in the original section.

Calculate the Transformed Section, I_{TR}

1. Use the ratio of the E modulus from each material to calculate a modular ratio, n.

   Transformed material

   $$n_A = \frac{E_A}{E_B} \quad \text{and} \quad n_C = \frac{E_C}{E_B}$$

   Base material

2. Usually the softer (lower E) material is used as a base (denominator). Each material combination has a different n.

3. Construct a transformed section by scaling the width of each material by its modular, n.

4. I_{tr} is calculated about the N.A.

5. If needed, separate transformed sections must be created for each axis (x-x and y-y)
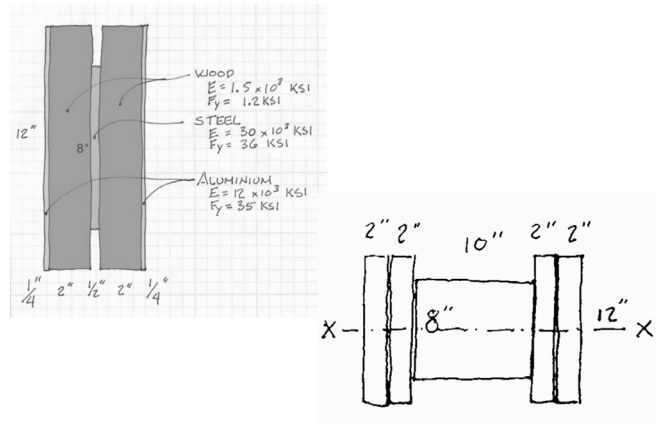
$$I_{tr} = \sum I + \sum Ad^2$$

Flitched Beam Analysis Procedure

1. Determine the modular ratio(s). Usually the softer (lower E) material is used as a base (denominator). Each material has a different n.

$$n_A = \frac{E_A}{E_B} \quad \text{and} \quad n_C = \frac{E_C}{E_B}$$

Base material

2. Construct a transformed section by scaling the width of the material by its modular, n.

3. Determine the Centroid and Moment of Inertia of the transformed section.

4. Calculate the flexural stress in **each** material separately using:

$$f_b = \frac{Mc\ n}{I_{tr}}$$

$$I_{tr} = \sum I + \sum A d^2$$

Transformation equation or solid-void

Analysis Example – pass / fail

For the composite section, find the maximum flexural stress level in each laminate material.
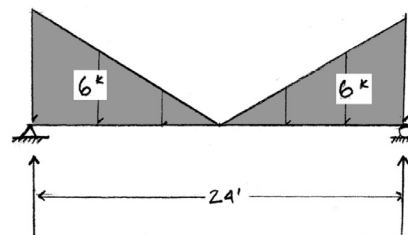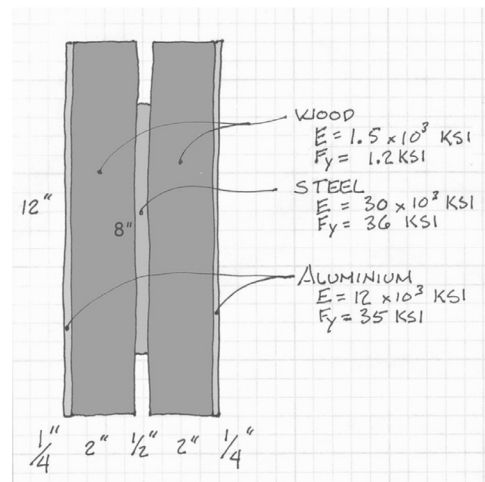
$$f_b = \frac{Mc\ n}{I_{tr}}$$

1. Determine the modular ratios for each material.

Use wood (the lowest E) as base material.

$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

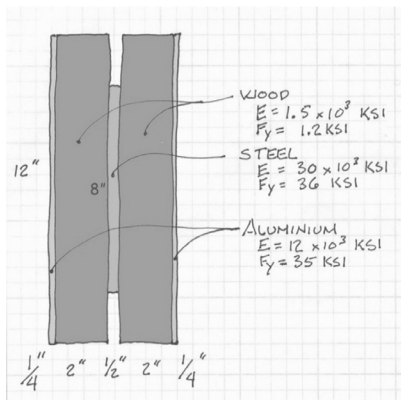$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$

LOADING DIAGRAM

Analysis Example cont.:

2. Construct a transformed section.

   Determine the transformed width of each material.

ALUM.

$t = \frac{1}{4}''$

$t_{tr} = \frac{1}{4} \times n_{AL}$

$= \frac{1}{4}(8.0) = 2.0''$

STEEL

$t = \frac{1}{2}''$

$t_{tr} = \frac{1}{2} \times n_{ST}$

$= \frac{1}{2}(20) = 10.''$

WOOD

$t = t_{tr} = 2''$

$n_{WOOD} = \frac{1.5}{1.5} = 1.0$

$n_{AL} = \frac{12}{1.5} = 8.0$

$n_{ST} = \frac{30}{1.5} = 20.$
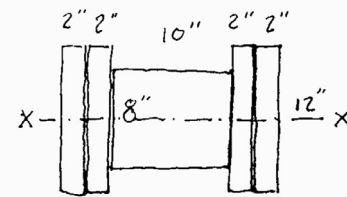
Transformed Section

Analysis Example cont.:

2. Construct a transformed section.

3. Calculate the Centroid and the Moment of Inertia for the transformed section.

$I_{tr} = \frac{8 \cdot 12^3}{12} + \frac{10 \cdot 8^3}{12}$

$= 1152 + 426$

$= 1578 \text{ in}^4$

Analysis Example cont.:

Find the maximum moment.

LOADING DIAGRAM

$$\sum M_{\mathcal{C\!L}} : 0 =$$
$$6(12') - 6(8') - M_{\mathcal{C\!L}} = 0$$
$$M_{\mathcal{C\!L}} = 24^{K-1}$$

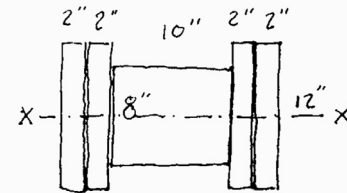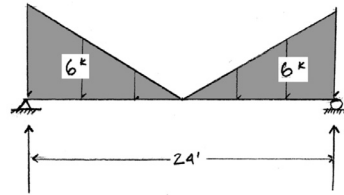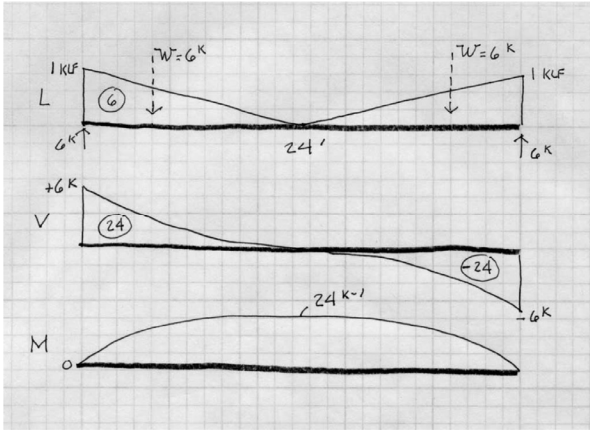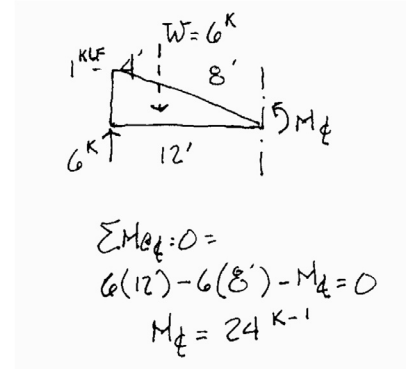by diagrams   or   by summing moments

---

Analysis Example cont.:

Actual Stresses:

4. Calculate the stress for each material using stress equation with the transformed moment of inertia.

$$f_b = \frac{Mc\,n}{I_{tr}}$$

Compare the stress in each material to limits of yield stress or the safe allowable stress.
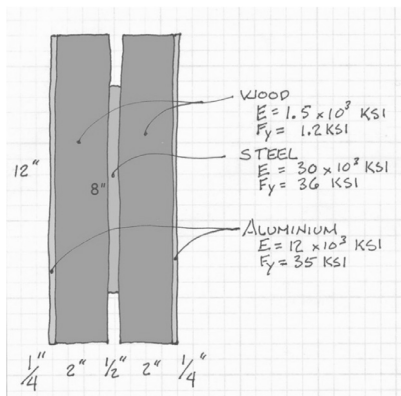
n = 12/1.5 = 8

$$f_{AL} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(6'')\,8}{1578}$$
$$= 8.76 \text{ KSI} \quad (f_y \approx 35 \text{ KSI})$$

n = 30/1.5 = 20

$$f_{ST} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(4'')\,20}{1578}$$
$$= 14.6 \text{ KSI} \quad (f_y \approx 36 \text{ KSI})$$

WOOD
E = 1.5 x 10³ KSI
Fy = 1.2 KSI
STEEL
E = 30 x 10³ KSI
Fy = 36 KSI
ALUMINIUM
E = 12 x 10³ KSI
Fy = 35 KSI

12" 8"

¼" 2" ½" 2" ¼"

n = 1.5/1.5 = 1

$$f_{WD} = \frac{Mc\,n}{I_{tr}} = \frac{24(12)(6'')\,1.0}{1578}$$
$$= 1.09 \text{ KSI} \quad (f_y \approx 1.2)$$

Capacity Analysis (ASD)
Flexure

Given
- Dimensions
- Material

Required
- Load capacity

1. Determine the modular ratio. It is usually more convenient to transform the stiffer material.

REDWOOD Nº 2
4 x 6 (3.5" x 5.5")
$F_b = 725$ psi   NDS 2015
$E = 1000$ KSI

A-36 STEEL
1/4" x 3.5"
$F = 21.6$ KSI
$E = 29000$ KSI

$$n = \frac{E_s}{E_w} = \frac{29000}{1000} = 29$$

Capacity Analysis (cont.)

2. Construct the transformed section. Multiply all widths of the transformed material by n. The depths remain unchanged.

3. Calculate the transformed moment of inertia, I_{tr}.

$$I_{tr} = \sum I + \sum Ad^2$$

3.5"   3.5"(29) = 101.5"   1/4"

NA

$$I_w = \frac{3.5(5.5)^3}{12} = 48.53 \text{ in}^4$$

$$I_s = 2\left[\frac{101.5(0.25)^3}{12} + 25.375(2.875)^2\right]$$
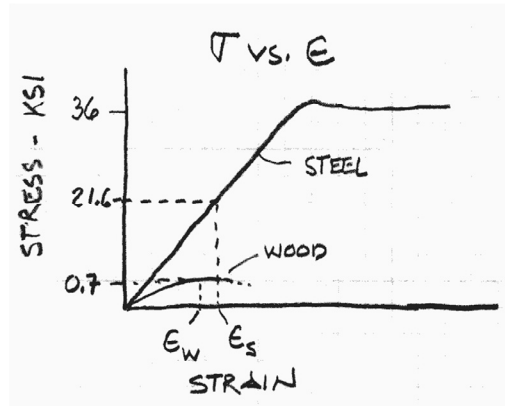
$$I_s = 2[0.132 + 209.74] = 419.7 \text{ in}^4$$

$$I_{TR} = 48.83 + 419.7 = 468.3 \text{ in}^3$$

Capacity Analysis (cont.)

4. Calculate the allowable strain based on the allowable stress for the material.

$$\varepsilon_{allow} = \frac{F_{allow}}{E}$$

$$\epsilon = \frac{\sigma}{E}$$

$$\epsilon_w = \frac{725}{1000000} = 0.000725$$

$$\epsilon_s = \frac{21.6}{29000} = 0.000745$$

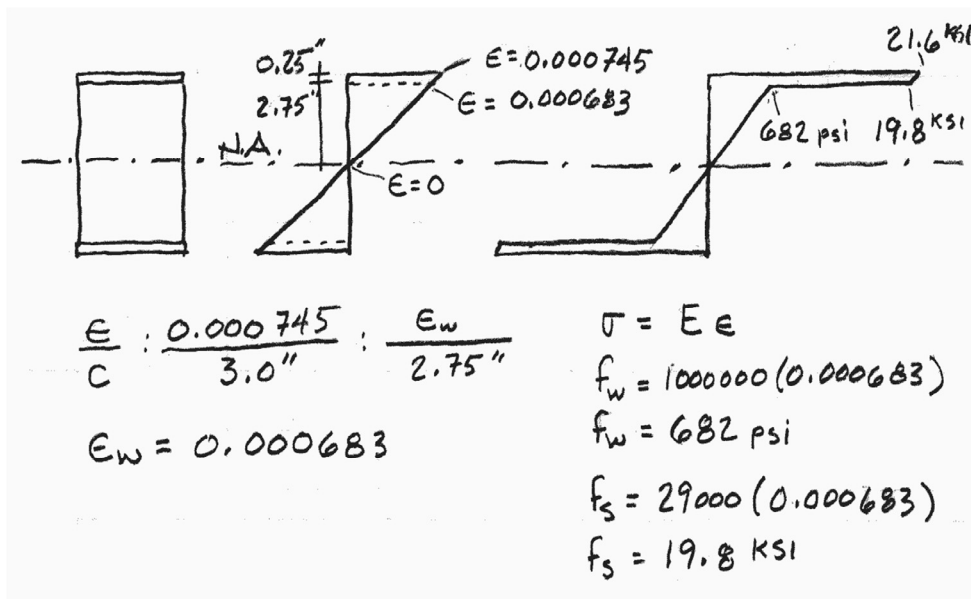---

Capacity Analysis (cont.)

5. Construct a strain diagram to find which of the two materials will reach its limit first. The diagram should be linear, and neither material may exceed its allowable limit.

Allowable Strains:

$$\epsilon = \frac{\sigma}{E}$$

$$\epsilon_w = \frac{725}{1000000} = 0.000725$$

$$\epsilon_s = \frac{21.6}{29000} = 0.000745$$

$$\frac{\epsilon}{c} : \frac{0.000745}{3.0"} ; \frac{\epsilon_w}{2.75"}$$

$$\epsilon_w = 0.000683$$

$$\sigma = E\epsilon$$

$$f_w = 1000000\,(0.000683)$$

$$f_w = 682 \text{ psi}$$

$$f_s = 29000\,(0.000683)$$

$$f_s = 19.8 \text{ KSI}$$

Capacity Analysis (cont.)

6. The allowable moments (load capacity) may now be determined based on the stress of either material. Either stress should give the same moment if the strain diagram from step 5 is compatible with the stress diagram (they align and allowables are not exceeded).

$$M_s = \frac{f_s\, I_{TR}}{c\,n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2\ ^{K\text{-}''}$$

$$M_w = \frac{f_w\, I_{TR}}{c} = \frac{0.682\,(468.3)}{2.75''} = 116.1\ ^{K\text{-}''}$$

7. Alternatively, the controlling moment can be found without the strain investigation by using the maximum allowable stress for each material in the moment-stress equation. The **lower momen**t will be the first failure point and the controlling material.

$$M_S = \frac{F_S\, I_{TR}}{c\,n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{\,K\cdot''} \leftarrow$$

$$M_w = \frac{F_w\, I_{TR}}{c} = \frac{.725\,(468.3)}{2.75''} = 123.5^{\,K\text{-}''}$$

Design Procedure:

Given: Span and load conditions
         Material properties
         Wood dimensions
Req'd: Steel plate dimensions

1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.
4. Based on strain compatibility with wood, find the largest d for steel where $\epsilon_s < \epsilon_{allow}$.
5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
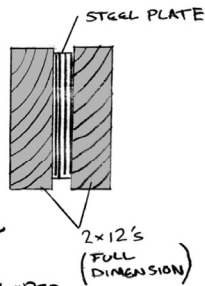7. Choose final steel plate based on available sizes and check total capacity of the beam.
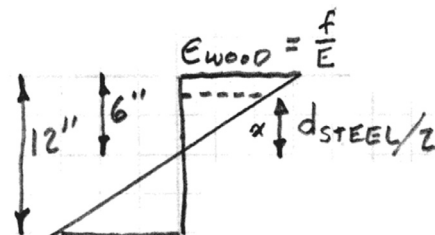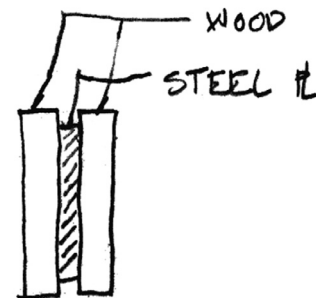
Design Example:
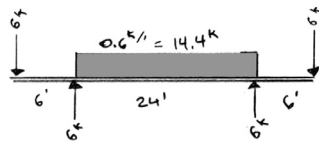
WOOD: $E = 2000$ KSI $f_{all} = 1.5$ KSI
STEEL: $E = 30000$ KSI $f_{all} = 18$ KSI

(A) DETERMINE THE DIMENSIONS OF THE STEEL PLATE REQUIRED FOR NEGATIVE MOMENT.

(B) DETERMINE THE LENGTH OF THE PLATES REQUIRED FOR NEGATIVE AND POSITIVE MOMENT.

2×12's (FULL DIMENSION)

1. Determine the required moment.
2. Find the moment capacity of the wood.
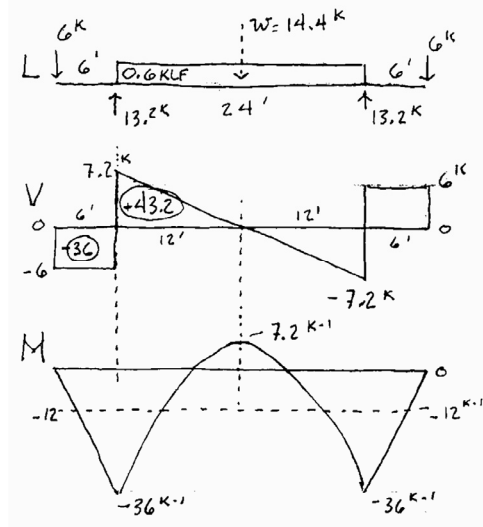3. Determine the required capacity for steel.

WOOD

$b = 2"$ $d = 12"$

$S_x = \dfrac{bd^2}{6} = \dfrac{2(144)}{6} = 48 \text{ in}^3$

$\times 2$ pcs. $S_{wood} = 96 \text{ in}^3$

$M_{WOOD} = F_b S_x$
$= 1.5 \text{ KSI } 96 \text{ in}^3 = 144 \text{ K-"}$
$= 12 \text{ K-I}$

$M_{TOTAL} = M_{WOOD} + M_{STEEL} = 36 \text{ K-I}$
$M_{STEEL} = 36 \text{ K-I} - 12 \text{ K-I} = 24 \text{ K-I}$

Design Example cont:
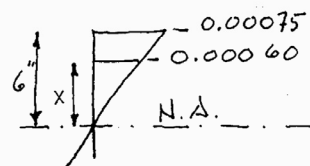
4. Based on strain compatibility with wood, find the largest d for steel where $\epsilon_s \leq \epsilon_{allowable}$.

ALLOWABLE STRAINS

$\epsilon_w = \dfrac{f}{E} = \dfrac{1.5}{2000} = 0.00075$

$\epsilon_s = \dfrac{f}{E} = \dfrac{18}{30000} = 0.00060$
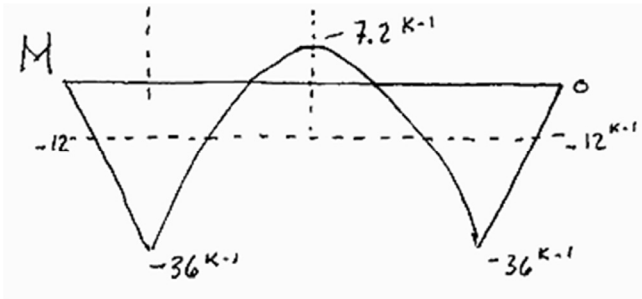
STRAIN DIAGRAM

$\dfrac{75}{6} : \dfrac{60}{x}$ $x = 4.8"$

$\therefore$ d STEEL PLATE = $9.6"$

Design Example cont:

5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.

STEEL

$M_{STEEL} = 24^{K-1} = 288^{K-11}$

$F_{ST} = 18^{KSI}$ (GIVEN)

$S'_x = \dfrac{M}{F} = \dfrac{288}{18} = 16 \text{ in}^3$

STEEL PLATE

$S_x \text{ REQ'D} = 16 \text{ in}^3 = \dfrac{bd^2}{6}$

$b = \dfrac{S_x \cdot 6}{d^2} = \dfrac{16(6)}{9.6^2} = 1.042''$

ROUND TO $\frac{1}{8}'' = 1\frac{1}{8}''$ (SAFE)

∴ USE
$9.5'' \times 1\frac{1}{8}''$
$S_x = 16.9 \text{ in}^3$

Design Example cont:
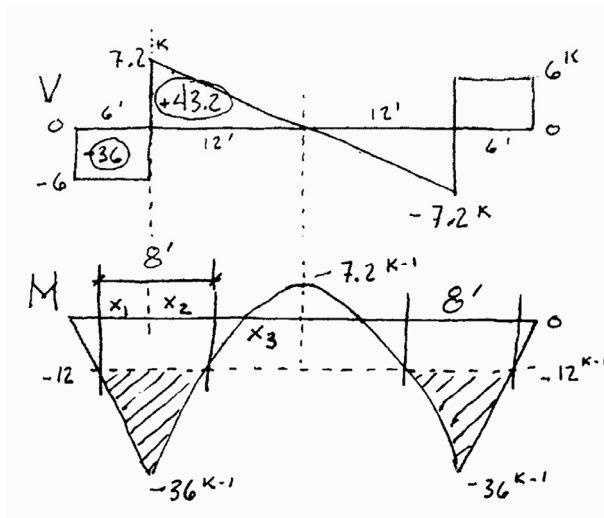
8. Determine required length and location of plate.

PLATE LENGTH

$\dfrac{36}{24} \quad \dfrac{6}{x_1} \qquad x_1 = 4'$

SHEAR AREA = 24

$43.2 - 24 = 19.2$

$\dfrac{bd}{2} = 19.2$

$\dfrac{x_3 \left(\frac{7.2}{12} x_3 \right)}{2} = 19.2$

$x_3^2 = 64 \quad x_3 = 8'$

$x_2 = 12 - x_3 = 4'$

∴ PLATE LENGTH = 8'

Applications:

Renovation in Edina, Minnesota

Four 2x8 LVLs, with two 1/2" steel
plates. 18 FT span
Original house from 1949
Renovation in 2006
Engineer: Paul Voigt

© Todd Buelow used with permission

Applications:

Renovation

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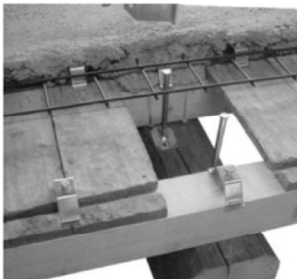
Steel Sandwiched Beams

Also based on strain compatibility

Wood – Concrete Composites

For Slabs or Slabs + Beams

(a)   (b)   (c)

Wood – Concrete Composites
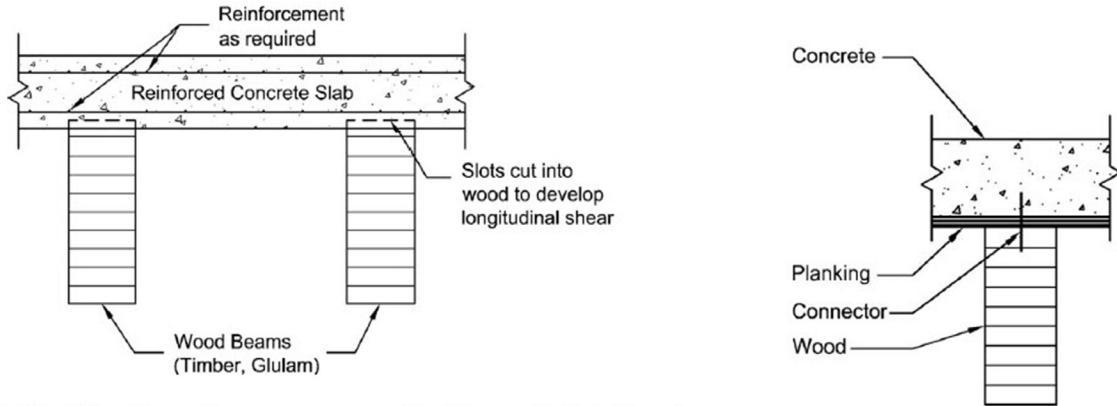
For Slabs or Slabs + Beams

Fig. 1. Traditional wood–concrete composite T-beam deck (adapted from Clouston and Schreyer 2006)

Wood – Concrete Composites
Comparison of connectors
from: Yeoh, David, et al., State of the Art on Timber-
Concrete Composite Structures: Literature Review,
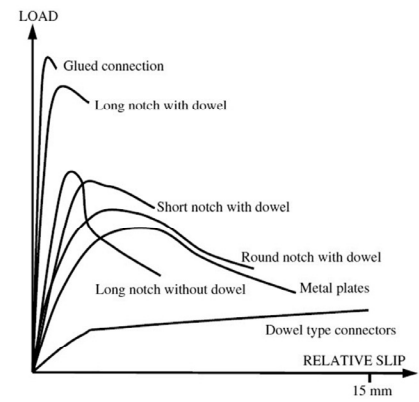*Journal fo Structureal Engineering*, ASCE Oct 2011

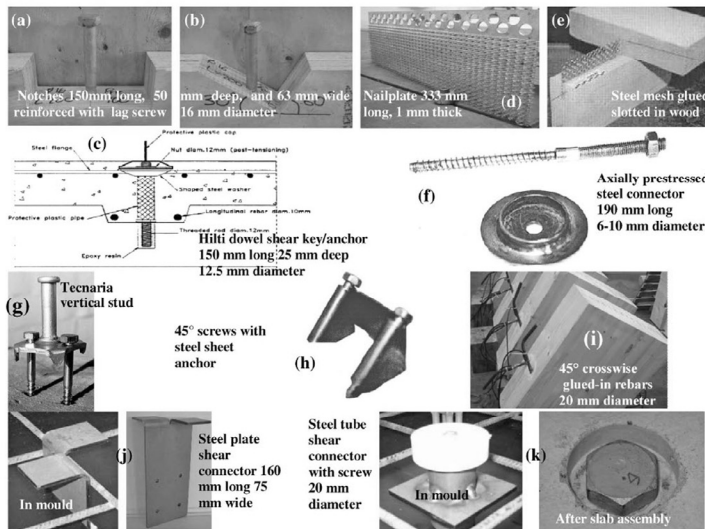**Fig. 1.** Comparisons of different categories of connection systems

Fig. 2. Different connection systems: (a), (b), and (d) reprinted from Yeoh (2010); (c) reprinted from Gutkowski et al. (2004, ASCE); (e) reprinted from Clouston et al. (2005, ASCE); (f) Fig. 4: Axial prestressed steel connector, reprinted from Capozucca (1998) with permission from RILEM; (g) Fig. 1: "Tecnaria" connector, reprinted from Fragiacomo et al. (2007a) with permission from RILEM; (h) reprinted from Steinberg et al. (2003, ASCE); (i) reprinted from Kuhlmann and Aldi (2008) with permission from author; (j) and (k) Fig. 9: Steel plate shear connector SP+N and Fig. 7: Steel tube shear connector SST+S, reprinted from Lukaszewska et al. (2008) with permission from RILEM